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Equivalence Between Passive Gravitational Mass And Energy For A Quantum Body

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Abstract— It is shown that passive gravitational mass operator of a hydrogen atom in the post-Newtonian approximation of the general relativity does not commute with its energy operator, taken in the absence of gravitational _eld. Nevertheless, the equivalence between the expectation values of passive gravitational mass and energy is shown to survive at a macroscopic level for stationary quantum states. Breakdown of the equivalence between passive gravitational mass and energy at a microscopic level for stationary quantum states can be experimentally detected by studying unusual electromagnetic radiation, emitted by the atoms, supported and moved in the Earth gravitational field with constant velocity.

Index Terms— Equivalence principle; General Relativity; Gravitational mass; Inequivalence; Mass-energy equivalence; Post-Newtonian approximation; Quantum gravity; Stationary quantum states; Time dependent oscillations; Unusual electromagnetic radiation.

1 INTRODUCTION

It is known that gravitational mass of a composite classical body in the general relativity is not a

trivial notion. For example, for two electrostatically bound objects with bare masses m_1 and m_2, only averaged over time gravitational mass, $\langle m^g \rangle t$, satisfies the Einstein equation [1], [2]:

$$\begin{aligned}\langle m^g \rangle t &= m_1 + m_2 + \langle K+U \rangle t + \langle 2K+U \rangle t \\ &= m_1 + m_2 + \frac{E}{c^2}\end{aligned} \quad (1)$$

where K is kinetic energy, U is potential energy, E is the total energy. It is important that the virial term in Eq. (1) is zero due to the virial theorem, $\langle 2K+U \rangle t = 0$.

2 GRAVITATIONAL MASS OF A QUANTUM BODY AT MACROSCOPIC LEVEL

The main goal of this paper in to consider a quantum problem about passive gravitational mass of a hydrogen atom in the Earth gravitational field. We define the gravitational mass as a quantity proportional to a weight of the atom in a weak centrosymmetric gravitational field [3]:

$$\begin{aligned}ds^2 &= -\left(1+2\frac{\phi}{c^2}\right)(cdt)^2 + \left(1-2\frac{\phi}{c^2}\right)\left(dx^2+dy^2+dz^2\right) \\ \phi &= -\frac{GM}{R}\end{aligned} \quad (2)$$

where $|\phi/c^2| \ll 1$, G is the gravitational constant, c is the velocity of light, M is the Earth mass, R is a distance from a center of the Earth and a center of mass of a hydrogen atom (i.e., proton).

For interval in Eq. (2), the efective Schrodinger equation for a hydrogen atom is derived in [4] to calculate the so-called "gravitational Start effect". Below, we consider completely diferent phenomena and have to use the Hamiltonian from [4] without tidal terms. As a result of disregarding all tidal terms, we obtain the following Schrodinger equation:

$$\hat{H} = m_e c^2 + \frac{\hat{p}^2}{2m_e} - \frac{e^2}{r} + \hat{m}_e^g \phi \quad (3)$$

where we introduce passive gravitational mass operator of an electron as:

$$\hat{m}_e^g = m_e + \left(\frac{\hat{p}^2}{2m_e} - \frac{e^2}{r}\right)\frac{1}{c^2} + \left(2\frac{\hat{p}^2}{2m_e} - \frac{e^2}{r}\right)\frac{1}{c^2} \quad (4)$$

where m_e is a bare electron mass, $\hat{p}$ is electron momentum operator, r is a distance between electron and proton. Supose that we have macroscopic ensemble of hydrogen atoms with each of them being in a ground state with energy E_1. Then, as follows from Eq. (4), the expectation value of the gravitational mass per one electron is:

$$\begin{aligned}\hat{m}_e^g &= m_e + \frac{E_1}{c^2} + \left\langle 2\frac{\hat{p}^2}{2m_e} - \frac{e^2}{r} \right\rangle \frac{1}{c^2} \\ &= m_e + \frac{E_1}{c^2}\end{aligned} \quad (5)$$

where the term in brackets is zero due to the quantum virial theorem [5]. Thus, we formulate our first result: the equivlence between passive gravitational mass and energy, taken in the absence of gravitational field, survives at a macroscopic level for stationary quantum states.

3 Gravitational Mass Of A Quantum Body At Microscopic Level

Here, we describe a thought experiment, which shows that Eq. (5) breaks the equivalence between passive gravitational mass and energy at a microscopic level, which is our second result. We consider the case, where gravitational field is adiabatically switched on, which corresponds to the following time-dependent perturbation:

$$\hat{U}(r,t) = \phi(R)\exp(\lambda t)\left[\frac{\left(\frac{\hat{p}^2}{2m_e} - \frac{e^2}{r}\right)}{c^2} + \frac{\left(2\frac{\hat{p}^2}{2m_e} - \frac{e^2}{r}\right)}{c^2}\right] \quad (6)$$

where:

$$\lambda \to 0$$

[Note that our choice of the perturbation in Eq. (6) allows to disregard all velocity dependent terms.] Suppose that, at $t \to \infty$ (i.e., in the absence of the field), a hydrogen atom is in its ground state:

$$\Psi_1(r,t) = \Psi_1(r)\exp\left(-i\frac{E_1 t}{\hbar}\right) \quad (7)$$

Then, at $t \to 0$ (i.e., in the presence of the field), the electron wave function can be written as:

$$\Psi(r,t) = \sum_{n=1}^{\infty} a_n(t)\Psi_n(r)\exp\left(-i\frac{E_n t}{\hbar}\right) \quad (8)$$

where $\Psi_n(r)$ are normalized s-type electron wave functions with energies E_n. The standard calculations show that the probability that, at $t=0$, an electron occupies n-th ($n \neq 1$) energy level is:

$$P_n = |a_n(0)|^2 = \left[\frac{\phi(R)}{c^2}\right]^2 \left[\frac{V_{(n,1)}}{(E_n - E_1)}\right]^2 \approx 0.49x10^{-18}\left[\frac{V_{(n,1)}}{(E_n - E_1)}\right]^2 \quad (9)$$

where $V_{(n,1)}$ is a matrix element of the virial operator $\hat{V}(r) = (2\hat{p}^2/2m_e) - (e^2/r)$.

$$V(n,1) = \int \Psi_n^*(r)\hat{V}(r)\Psi_1(r)d^3r \qquad \hbar\omega_{(n,1)} = \frac{(E_n - E_1)}{\hbar} \quad (10)$$

for $n \neq 1$.

4 Gravitational Mass Of A Quantum Body At Microscopic Level

[Here, we use $M \approx 6x10^{24}$ kg, $R \approx 6.4x10^6$ m.] In fact, this means that a measurement of the gravitational mass (4) gives the following quantized values:

$$m_e^g(n) = m_e + \frac{E_n}{c^2} \quad (11)$$

Instead of the expected Einstein equation, $m_e^g = m_e + E_1/c^2$. It is important that the excited energy levels spontaneously decay and, thus, the quantization law of Eq. (11) can be deteced by measuring electromagnetic radiation, emitted by macroscopic ensemble of hydrogen atoms.

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